Symmetry and Group Theory – Exercise Set 3

3.1) Show that all molecules are chiral that do not possess an S_n axis.

3.2) Determine the point group of 1,3,5,7-tetramethylcyclooctatetraene. Show that the molecule is achiral even though it does not have a reflection plane or a center of inversion.

3.3) Sketch the dipole moment in NH_3, phenol, PPh_3.

3.4) Show that the group multiplication table of $C_{2v} = \{E, C_2, \sigma_v(xz), \sigma_v(yz)\}$ is the one given below. Show this by expressing the symmetry operations by transformation matrices acting on a point in Cartesian coordinates $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

C_{2v}	E	C_2	$\sigma_v(xz)$	$\sigma_v(yz)$
E	E	C_2	$\sigma_v(xz)$	$\sigma_v(yz)$
C_2	C_2	E	$\sigma_v(yz)$	$\sigma_v(xz)$
$\sigma_v(xz)$	$\sigma_v(xz)$	$\sigma_v(yz)$	E	C_2
$\sigma_v(yz)$	$\sigma_v(yz)$	$\sigma_v(xz)$	C_2	E

3.5) Show that $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

3.6) Are the natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$ a group with respect to multiplication?

3.7) Are the integers $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ a group with respect to addition?

3.8) Name subgroups of C_{2v}.

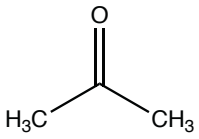
3.9) A , B, and C are elements of a group. Prove that if A is conjugate with B and A is conjugate with C, then B is conjugate with C.

3.10) Determine the multiplication table of the group G_3 = {E, A, B}. What can you tell about the properties of this group? Determine the classes of G_3.

Homework

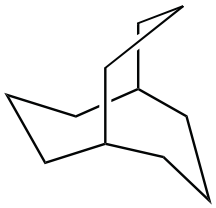
3.11) Determine the point groups of the following molecules and objects.

A

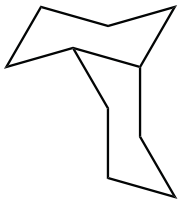


acetone,
treat methyl groups
as spheres

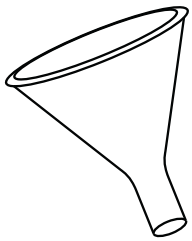
B



C

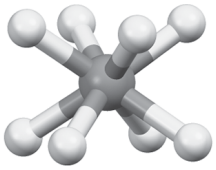


D



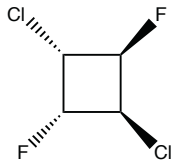
E

F



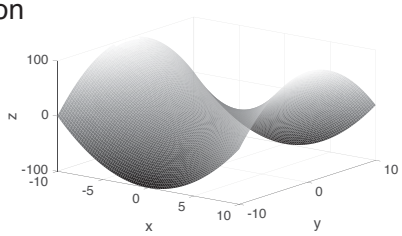
$[XeF_8]^{2-}$
(square antiprism)

G



H

surface described
by the function
$z = x^2 - y^2$



3.12) Construct the group multiplication table of the point group C2h. Determine the classes of the point group C2h.

3.13) Which point group is obtained if one deletes the inversion operation i from the point group S6?

3.14) Show that the numbers $\{c, c^2, c^3, \dots, c^n\}$ with $c = e^{i\frac{2\pi}{n}}$ and integer n form a group with respect to multiplication.

3.15) Show that $G = \{1, -1, i, -i\}$ is a group with respect to multiplication. Here, i refers to the complex number with $i^2 = -1$.

a) First, write down the group multiplication table.
b) Then show that G meets all the criteria for the definition of a group.
c) Which special properties does the group G have?